**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans : 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44

Ans: Probability that the employee will be greater than age of 44 =100-84.1345=15.86%

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|  |  | So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345% |
|  |  | Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE. |

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : pr(x<30)=Pr(z<(30-38/6)=Pr(z<1.333)=9.12%

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|  | So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees). |
|  | Therefore the statement B of the question is also TRUE. |

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *id* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans : As we know that if x ~N(µ1, σ1^2),and Y ~N(µ2, σ2^2) are two independent random variables then x + Y ~ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )

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|  | Similarly if Z = ax + by , where X and Y are as defined above, i.e. Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ). |
|  | Therefore in the question |
|  | 2X1~ N(2 u,4 σ^2) and |
|  | X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) |
|  | 2X1-(X1+X2) = N(4µ,6 σ^2) |
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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: 48.5,151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans : See Jupyter Notebook